QUESTION 1

Let:

$$\begin{aligned} A &= \tan(22.5^\circ) \\ B &= \csc(22.5^\circ)\sec(22.5^\circ) \\ C &= \cot(67.5^\circ) \end{aligned}$$

Find $AB - AC$.

QUESTION 2

Let:

$$\begin{aligned} A &= \begin{vmatrix} 1007 & 1005 \\ 1008 & 1006 \end{vmatrix} \\ B &= \text{the period of the function } y = 6\tan\left(\frac{\pi x}{5}\right) + 3 \end{aligned}$$

Evaluate $\displaystyle\lim_{n\to\infty}\left(1+\frac{A}{n}\right)^{Bn}$.

QUESTION 3

Let:

$$
\begin{aligned}
A &= \text{in the complex plane, the complex number equidistant to } -3+i, 11+7i, \text{ and } 11+i \\
B &= \left|\frac{3+4i}{8+15i}\right| \\
C &= \sum_{n=1}^{5} \ln\left(\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^n\right) \text{ over the domain of complex numbers} \\
D &= \text{the number of solutions of } x^{13}-1=0 \text{ in the 2nd quadrant of the complex plane}
\end{aligned}
$$

Find $17AB + 2C + D$.

QUESTION 4

Let:

$$A = \text{the eccentricity of the conic } r = \frac{6}{1 + 3\cos(\theta)}$$

$$B = \text{the number of intersections between } r^2 = 4\sin(2\theta)\text{, and } r = 2 \text{ over the interval } 0 \le \theta \le 2\pi$$

$$C = \text{the number of petals in the polar curve } r = 4\sin(5\theta)$$

D = A bug crawls the curve outlined by the equation $r = \sin(\theta)$.
Let D be the distance the bug travels over the interval $0 \le \theta \le \pi$.

Find: $A + B + C + \dfrac{D}{\pi}$.

QUESTION 5

Let:

$$
\begin{aligned}
A &= \text{the length of the major axis of the conic: } 4x^2 - 16x + 9y^2 + 18y - 11 = 0 \\
B &= \text{the length of the latus rectum of the conic: } 4x^2 + 8x + 4 + 4y^2 = 16 \\
C &= \text{the least angle of rotation of axes needed to eliminate the cross-term of the conic:} \\
  & \quad 13x^2 - 10xy + 13y^2 - 34x\sqrt{2} + y\sqrt{2} - 22 = 0 \\
D &= \text{the focal radius of the conic: } 16x^2 - 9y^2 - 32x - 36y - 36 = 128
\end{aligned}
$$

Find $A + B + 12C + D$.

QUESTION 6

Let:

$$\begin{aligned}
A &= \text{the sum of the cubes of the roots of } f(x) = x^3 - 2x^2 + 5x - 6 \\
B &= \text{the number of complex roots of the function } f(x) = x^5 + 4x^6 - 3x^2 + 2 \\
C &= \sum_{n=0}^{\infty} \frac{n^2 + 8n + 7}{2^n} \\
D &= f^{\infty}(1), \text{ given the function } f(x) = \frac{1}{2}x + 1 \text{ and } f^n(x) = f(f(\ldots)), \, n \text{ times}
\end{aligned}$$

Find $A + B + C + D$.

QUESTION 7

Let:

$$\begin{aligned}
A &= \text{the magnitude of the cross product of the vectors } \langle 3,0,4\rangle \text{ and } \langle 1,2,4\rangle \\
B &= \text{the dot product of the vectors } \langle 1,0,-2\rangle \text{ and } \langle 3,4,1\rangle \\
C &= \text{the sum of the components of } \overrightarrow{AC}\text{, given } \overrightarrow{AB} = \langle 3,2\rangle \text{ and } \overrightarrow{CB} = \langle 4,3\rangle \\
D &= \text{the cosine of the angle inscribed between the vectors } \langle 1,3,2\rangle \text{ and } \langle 3,0,-3\rangle
\end{aligned}$$

Find $A + B + C + 14D$.

QUESTION 8

Let:

$$\begin{aligned} A &= \text{the distance between the polar coordinates } \left(3, \frac{\pi}{8}\right), \text{ and } \left(4, \frac{7\pi}{8}\right) \\ B &= \text{the volume of the parallelepiped determined by } \langle 1, 1, 2\rangle\ , \langle 3, 0, -1\rangle, \text{ and } \langle 2, -1, 1\rangle \\ C &= \text{the largest area of a triangle given two sides of the triangle are both of length 3} \\ D &= \langle 1, 1, 3\rangle \bullet \langle 0, 3, -1\rangle \end{aligned}$$

Find $A^2 + B + 2C + D$.

QUESTION 9

Let:

$$
\begin{aligned}
A &= \text{the period of the function } f(x) = \sin\left(\frac{2}{3}x\right) + \cos(3x) \\
B &= \frac{\sin(1^\circ) + \sin(2^\circ) + \ldots \sin(n^\circ) \ldots + \sin(89^\circ)}{\cos(1^\circ) + \cos(2^\circ) + \ldots \cos(n^\circ) \ldots + \cos(89^\circ)} \\
C &= \text{the value of } \frac{4\cos(2\theta)}{1 + \sin(2\theta)}, \text{ when } \cos^2(\theta) - \sin^2(\theta) = 1 \\
D &= \prod_{n=1}^{359} \cos(n^\circ)
\end{aligned}
$$

Find $A + B + C + D$.

QUESTION 10

Let:

$$\begin{aligned} A &= \text{the value of the only real root to the equation } (x-10)^{20} = x^{20} \\ B &= \text{the value of the largest coefficient of the polynomial when } \left(\sum_{n=0}^{30} x^n\right)^2 \text{ is expanded} \\ C &= \text{the sum of the 14th powers of the roots of the equation } f(x) = x^{15} + 4x^2 - 2x \\ D &= \text{the sum of the reciprocals of the roots of the equation } g(x) = 2x^3 - 3x^2 + 4x + 8 \end{aligned}$$

Find $A^{\frac{-1}{D}} + B - C$.

QUESTION 11

Let:

$$A = \det\left(\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}^{2016}\right)$$

$$B = \det\left(\begin{bmatrix} 2 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^{-1}\right)$$

$$C = \sum_{n=0}^{\infty} \det\left(\begin{bmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{bmatrix}^{n}\right)$$

Find $A + 105BC$.

QUESTION 12

Let:

$$\begin{aligned}
A &= \cos(\theta), \text{ when } \frac{\tan\left(\dfrac{\theta}{2}\right)}{1-\tan^2\left(\dfrac{\theta}{2}\right)} = \frac{3}{4} \\
B &= \sin^2(10^\circ) + \sin^2(20^\circ) + \ldots + \sin^2(80^\circ) + \sin^2(90^\circ) \\
C &= \sin(15^\circ) + \cos(30^\circ) + \sin(60^\circ) + \tan(\pi) + \sin\left(\frac{13\pi}{12}\right) \\
D &= \text{the value of } \csc(\theta), \text{ when } \tan(\theta) + \cfrac{1}{\tan(\theta) + \cfrac{1}{\ldots}} = 2
\end{aligned}$$

Find $AC^2D + B$.

QUESTION 13

Let:

$$
\begin{aligned}
A &= \text{the greatest integer less than } \left(\sqrt{(9+2\sqrt{18})}\right)^6 \\
B &= \text{the remainder when } 37! \text{ is divided by } 38
\end{aligned}
$$

Find $A - B$.

QUESTION 14

Let:

$$A = \begin{vmatrix} 3 & 2 & 1 & 8 \\ 2 & -1 & 5 & 2 \\ 0 & 0 & 0 & -1 \\ 2 & 4 & 6 & 8 \end{vmatrix}$$

$$B = \text{the determinant of matrix } B \text{ in the equation } B\begin{bmatrix} 4 & 1 \\ 3 & 4 \end{bmatrix} - 2B = \begin{bmatrix} 3 & 0 \\ 4 & 2 \end{bmatrix}$$

Find $\dfrac{A}{B}$.